TM16

EXPERIMENTS IN VIBRATION
USING THE UNIVERSAL VIBRATION APPARATUS

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UNIVERSAL VIBRATION APPARATUS TM 16

SECTION ONE

1. Introduction

The Universal Vibration apparatus enables a comprehensive range of vibration experiments to be performed with the minimum amount of assembly time and the maximum of adaptability.

The experiments have been so designed as to lead the student through the basics of vibration theory by means of, initially, very simple experiments - which make way for those of a more extensive nature as experimental aptitude increases.

Although the policy of the experiments is to give the student a general insight into experimental methods, some attempt has been made to evoke further study and critical appreciation by subsidiary questions posed at the end of some of the tasks.

PURPOSE OF THE MANUAL

This manual has been written primarily to give details of the apparatus required and the experimental techniques involved for each experiment in turn. Each experiment is presented with an 'Introduction' dealing with the purpose and basic theory involved. Further sections detail the apparatus and experimental method with reference to photographs and diagrams included in the text.

Finally, the form of calculations and results is given, followed by any "Further Considerations" which may be significant.

SECTION TWO

2. GENERAL DESCRIPTION OF APPARATUS

2.1 Portal Frame

The apparatus consists of a basic portal frame, robustly constructed, from 68 mm square, rolled hollow section, vertical uprights and double 100 mm x 50 mm channel horizontal members. The frame is mounted on 4 castors for ease of mobility.

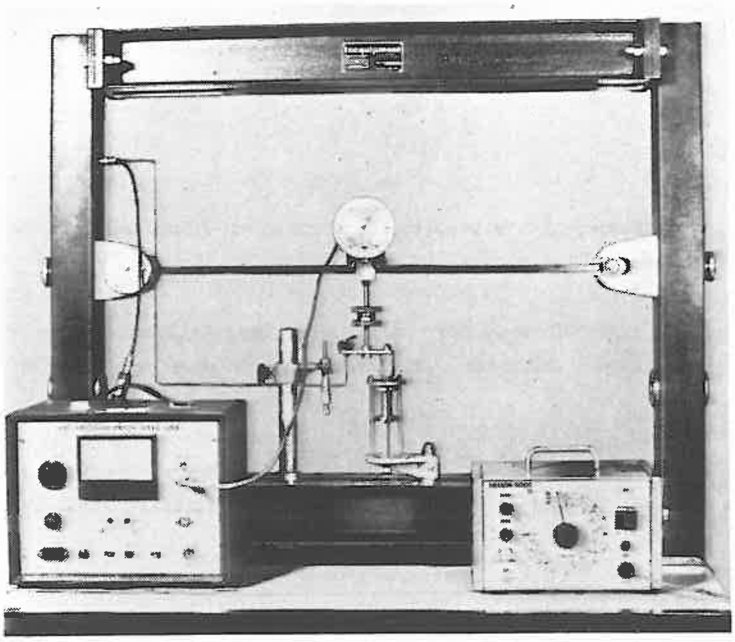
Screw jacks allow the weight of the frame to be transferred to the floor during operation of experiments, which enables the entire rig to be levelled prior to the experimental work and guarantees rigidity.

The frame has been fully machined so as to be adaptable to accept all the listed experiments. An attractive storage cupboard is fitted at the front which houses all the components when not in use.

2.2 E. 11 High Precision Speed Control Unit

A d.c. motor is used for all forced vibrations experiments powered by the E11 control unit (see below). This unit comprising control box and d.d. motor, provides high precision speed control of the motor up to 3000 rev/min , irrespective of the normal load fluctuations of the motor.

The front panel of the unit contains a speed control knob, a fully calibrated speed meter incorporating an automatic range switching device (there being two ranges: 0 - 1500 and 0 - 3000 rev/min, and power sockets for (i) mains input; (ii) d.c. motor; (iii) auxiliary output (either to a stroboscope or chart recorder).



The above photograph also show an oscilloscope (Not Supplied)which has been susperseded by the the TecQuipment E21. (See also Fig 10.1).

2.3 List of Components

Part No:	DESCRIPTION	EXPERIMENTS
B1	Steel Sub-frame (cross-beam)	1,2,3,4,5
B2	Wooden Ball for Pendulum	1
B3	Steel ball for Pendulum	1
B4	Kater (adjustable)pendulum	4
B5	Wooden compound pendulum	3
B6	Simple bob pendulum	2,3,
B7	Bifilar suspension (+ masses)	5
C1	Spring support	6,12,13,14
C2	Guide bush assembly	6
C3	Loading platform	6
D1	Trunnion mounting	10,11,12,12,14
D2	Dashpot assembly	10,11,12,13,14
D3	Dashpot bracket	12,13,14
D4	Out-of-balance discs	12,13,14
D5	Beam support	12,13,14
D6	Stylus and support	12,13,14
D7	Chart recorder	12,13,14
D8	Pivot support for stylus	14
D9	Beam clamp	12,13,14
E1	Trunnion mounting with lateral movement	10,11
E2	Support for dashpot	10,11
E3	Support for micrometer	10,11
E11	Speed control unit with exciter motor, graduated discs	10,11
E5	Contactor	10,11
E6	Rectangular section beam	10,11,12,13,14
G1	Vibration absorber	11

H1	Rotor (254 mm diameter)	7,9
H2	Rotor and additional masses (168 mm diameter)	9
I1	Bracket	7
K1	Shaft support bracket	8
K2	Dashpot assembly	8
K3	Rotor and recording drum	8
K4	Transparent oil reservoir	8

SECTION THREE

EXPERIMENT 1: SIMPLE PENDULUM

1. Introduction

One of the simplest examples of free vibration with neglible damping is the simple pendulum. The motion is simple harmonic, characterised by the equation

$$\frac{d^2x}{dt^2} = -\left(\frac{g}{l}\right) x$$

The periodic time is given by $\tau = 2\pi\sqrt{\frac{l}{g}}$

In this experiment, the object is to analyse the above equation for the periodic time by varying l (the length of the pendulum) and timing the oscillations. The independance of the size of the mass of the particle is also demonstrated.

2. Apparatus as shown in Fig 1.1

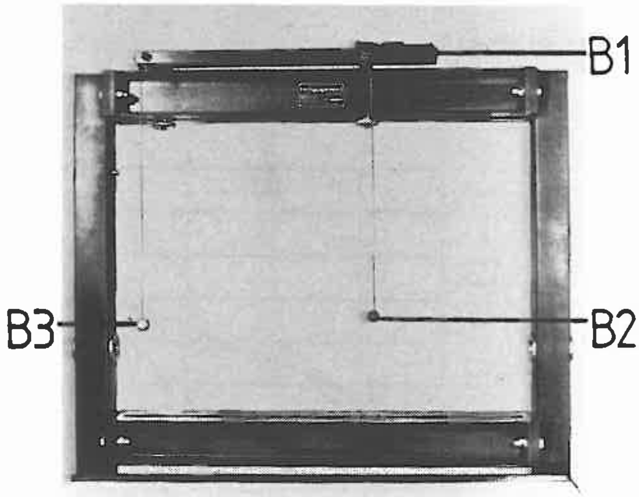


Fig.1.1

Sub-frame (cross-beam)	B1
Small Wooden Ball	B2
Small Steel ball	B3

Inextensible flexible cord (not supplied)
Stop watch or clock (not supplied)
Metre rule (not supplied)

Both the steel and wooden balls are attached to lengths of cord about 1 metre long each of the two cords being supended from the small chucks at either end of the sub-frame. By pulling the thread through the chuck and the hole above the sub-frame, the length can be varied.

3. Experimental procedure

Use a metre rule (not supplied) to measure the length 'l', the distance from the bottom of the chuck to the centre of the ball. The pendulum is displaced through a small angle and allowed to swing freely.

The time taken for, say, 50 complete oscillations is noted and the periodic time τ recorded.

This is repeated for various values of '1' for both the wooden ball and the steel ball. The results are entered in the table Fig 1.2. Values of τ^2 are then plotted against values of 1. The resulting graph is as shown in Fig 1.3

Length L (m)	Time for 50 complete oscillations		Period τ steel	τ^2 steel
	steel	wood		
0.10				
0.15				
0.20				
0.25				
0.30				
0.35				
0.40				
0.45				
0.50				

Fig 1.2 Table of results for Experiment 1

The graph results in a straight line giving a relationship between τ^2 and 1 of the form $\tau^2 = K.1$, where K is a constant equal to

$\frac{4\pi^2}{g}$ which is equal to the slope of the line.

Hence the value of g, the acceleration due to gravity can be determined.

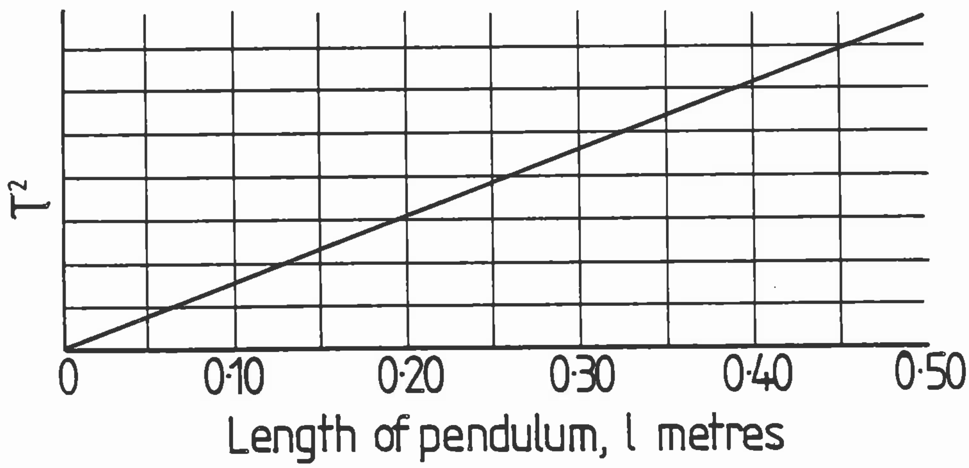


Fig 1.3 Graph of τ^2 against 1 for a simple pendulum

4. Further Considerations

1. What inaccuracies exist in this method for calculating a satisfactory value for 'g'?
2. How can one overcome these inaccuracies?

EXPERIMENT 2 COMPOUND PENDULUM

1. Introduction

A rigid body swinging about a fixed horizontal axis (see Fig 2.1), displaced through an angle θ, is subjected to restoring couple. mgh sin θ

If angle θ is sensibly small, the equation of motion becomes $\frac{d^2\theta}{dt^2} + \left(\frac{mgh}{I_A}\right)\theta = 0$

where m = mass of the body
h = distance of the mass centre from the swing axis
θ = angular displacement
I_A = moment of inertia of the body about the swing axis

The motion is simple harmonic; the constant $\frac{mgh}{I_A} = \omega^2$

and the periodic time $\tau = \frac{2\pi}{\omega}$

ie $\tau = 2\pi\sqrt{\frac{I_A}{mgh}}$

Now $I_A = I_G + mh^2$ (by the parallel axis theorem) and $I_G = mk^2$ where k is the radius of gyration of the body about axis through the mass centre parallel to the swing axis.

Therefore, $\tau = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$

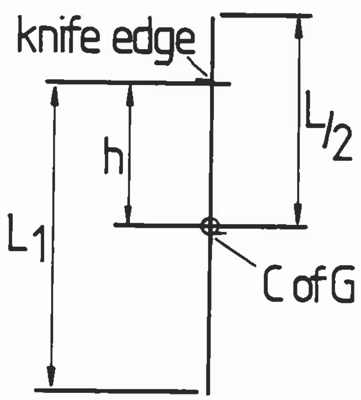


Fig 2.1

2. Apparatus

The compound pendulum consists of a 12.7 mm diameter steel rod 0.762 m long. The rod is supported on the cross member B1 by an adjustable knife-edge which, when moved along the rod, effectively alters the value of 'h' discused above.

3. Method

The centre of gravity of the rod is first located, (midway along the rod). The knife-edge is tightened at a given value of L_1 from one end and the rod is suspended by placing the knife-edge on the cross beam so that it swings freely through a small angle without any rotation at the support. The time for, say 50 oscillations is then noted and the periodic time τ recorded.

In order to perform a subsequent test, the knife-edge is slackened off and moved along the rod to a new position. It is found most convenient to remove the pendulum from the cross beam and to do any adjustments away from the portal frame.

The expression for the periodic time can be transformed to $\tau^2 h = \frac{4\pi^2}{g} \cdot h^2 + \frac{4\pi^2}{g} \cdot k^2$

Plotting a graph of $\tau^2 h$ to a base of h^2 a straight line is obtained. From the slope of the line g is found and from the intercept, k is determined and compared with the theoretical value.

4. Results

Tabulate the results as shown in Fig 2.2

Theoretical value of k can be found using Routh's Rule which for a rod of small cross-section gives $k^2 = L^3/3$.

L_1 (m)	h (m)	Time for 20 oscillations	Period τ (s)	h^2	$\tau^2 h$
0.45					
0.50					
0.55					
0.60					
0.65					
0.70					
0.75					

Fig 2.2

5. Further Considerations

1. Calculate the length of the simple equivalent pendulum for the above case $\tau = 2\pi\sqrt{\frac{l}{g}}$ (simple pendulum) is equal to $2\pi\sqrt{\frac{k^2 + h^2}{gh}}$

for a compound pendulum.

2. Find the two values of h which satisfy the resulting quadratic equation giving equal periodic times.

EXPERIMENT 3: CENTRE OF PERCUSSION

1. Introduction

If a compound pendulum supported on a horizontal pivot is subjected to an impact force at an arbitrary point, there will, in general, be a horizontal reaction at the pivot. The situation can be likened to a cricket bat striking a ball. There is one particular point at which the strike occurs, for which there is no horizontal reaction at the pivot of the compound pendulum; such a point is called the centre of percussion. The location of such a point is the object of this particular experiment.

The apparatus illustrated in Fig 3.1 consists of a steel ball as part of a simple pendulum (B6) and the rectangular shaped wooden compound pendulum (B5) having an adjustable steel weight slideable in a central slot. Both are suspended on steel knife-edges from the horizontal cross-beam (B1) at the top of the portal frame. The simple pendulum is located in a vee groove whilst the knife-edge of the compound pendulum rests on the flat surface of the beam.

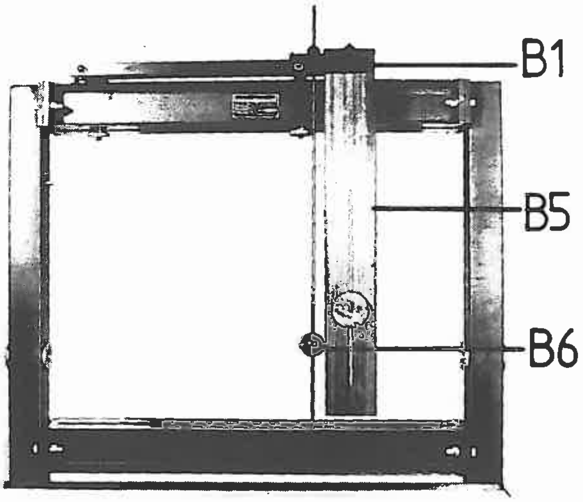


Fig 3.1

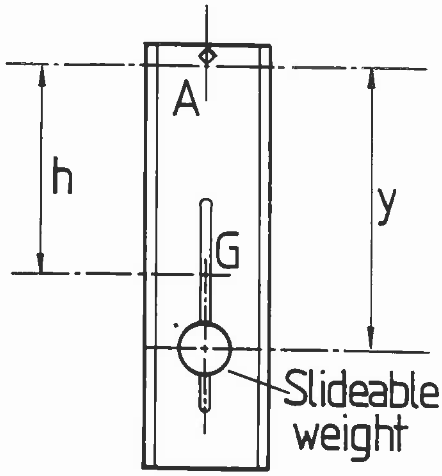


Fig 3.2

EXPERIMENT 3(i)

3. Method

To determine the centre of percussion of the compound pendulum, first of all determine its periodic time and then using the formula:

$$\tau = 2\pi\sqrt{\frac{k^2_A}{gh}} \qquad \ldots 3.1$$

determine k_A the radius of gyration about the pivot axis.

$$k^2_A = k^2 + h^2 \quad \text{(parallel axis theorem).}$$

h is the distance from the point of suspension to the centre of gravity.

The centre of gravity of the pendulum is determined by resting the board, with the steel weight at a distance y from a knife-edge support. The distance h from the knife-edge of the pendulum to the balancing knife edge then determined (see Fig 3.2).

For each position of the steel weight, the periodic time for 20 oscillations is determined. From the values of τ and h in equation 3.1, the value of K_A is then determined and compares with theoretical values.

Record all results in a table in the form of Fig 3.3.

Results

The results entered in the table in Fig 3.3 will indicate the variation of periodic time as the radius of gyration about the point of suspension varies. A theoretiical value for k, the radius of gyration about the centre of gravity may be calculated from the dimensions of the pendulum.

Test No	Time for 20 oscillations	Period τ (s)	y (m)	h (m)	k_A (m)	k (m)
1.						
2.						
3.						
4.						
5.						

Fig 3.3

EXPERIMENT 3(ii)

4. Using the results of Experiment 3(i), the centre of percussion may now be shown to be at a distance from the point of suspension equal to its equivalent length.

$$l = \frac{k^2 + h^3}{h}$$

where l is the length of the equivalent pendulum

k is the radius of gyration about the centre of gravity $k^2_A = k^2 + h^2$

h is the distance of the point of suspension from the centre of gravity

The experiment is performed by adjusting the length of the simple pendulum (B6) so that the length of the bob from the knife-edge is equal to the equivalent length of the compound pendulum determined above. The simple pendulum is allowed to swing, so that the spherical bob strikes the edge of the compound pendulum at its perigee (lowest point of its path) and causes the latter to swing.

By constraining horizontal movement of the simple pendulum in its vee groove, the only horizontal movement possible is that of the compound pendulum resting on its flat support. It may be observed that no horizontal movement is produced with the simple pendulum = l and that for any other values, horizontal movement is produced. (A pencil mark on the cross beam under the initial knife-edge position may be used as a reference mark).

EXPERIMENT 4

DETERMINATION OF THE ACCELERATION DUE TO GRAVITY BY MEANS OF A KATER (REVERSIBLE) PENDULUM

1. Introduction

The Kater pendulum is a device for accurately determining the acceleration due to gravity. This consists of two adjustable knife-edges and an adjustable cylindrical bob. By arranging their relative positions to give equal periodic times when suspended from either knife edge, two simultaneous equations are produced.

i.e. $$\tau_1 = 2\pi\sqrt{\frac{h_1^2 + K^2}{gh}} \quad \text{and} \quad \tau_2 = 2\pi\sqrt{\frac{h_2^2 + k^2}{gh_2}}$$

then $$\frac{gh_1\,\tau_1^2}{4\pi^2} = h_1^2 + K^2 \quad \text{and} \quad \frac{gh^2\,\tau_2^2}{4\pi^2} = h_2^2 + K^2$$

Then, by arrangement: $$\frac{4\pi^2}{g} = \frac{\tau_1^2 + \tau_2^2}{2(h_1 + h_2)} + \frac{\tau_1^2 - \tau_2^2}{2(h_1 - h_2)}$$

2. Apparatus

The apparatus required for this experiment is a pendulum having two adjustable knife edges and an adjustable cylindrical bob (B4) suspended from the hardened steel cross beam (B1). See Fig 4.1.

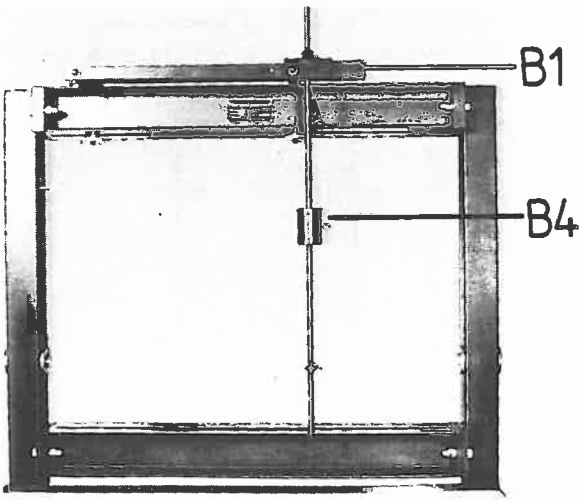


Fig 4.1

3. Method

The knife-edges are positioned a given distance apart, and the pendulum is then suspended from one of the knife-edges. The periodic time τ is then found by timing 50 oscillations. The pendulum is then reversed and suspended from the other knife-edge and by suitable positioning of the cylindrical bob, the periodic time τ_2 is obtained so as to be approximately equal to τ_1.

τ_1 is then rechecked and further adjustments made to obtain an equal time of swing.

Proceeding thus, τ_2 is obtained approximately equal to τ_1 after which τ_1 and τ_2 are determined for 200 swings. The centre of gravity of the final arrangement is then found by balancing on a knife-edge and h_1 and h_2 are determined, being the respective distances of the two knife-edges from the centre of gravity. The distance between the two knife-edges 'L' is thus the length of the simple equivalent pendulum.

4. Results

$h_1 = 0.20$ m $h_2 = 0.30$ m

$\tau_1 =$ $\tau_2 =$

$$\frac{4\pi^2}{g} = \frac{(\tau_1^2 + \tau_2^2)}{2(h_1 + h_2)} + \frac{(\tau_1^2 + \tau_2^2)}{2(h_1 - h_2)}$$

From which the value of g is determined

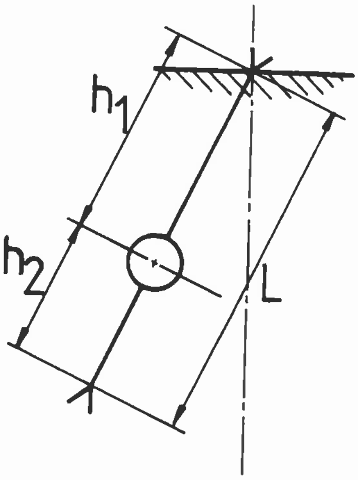


Fig 4.2

EXPERIMENT 5: BIFILAR SUSPENSION

1. Introduction

The bifilar suspension can be used to determine the moment of inertia about an axis through the mass centre of bodies which can be conveniently suspended by two parallel cords of equal length (see Fig 5.1). Angular displacement of the body about the vertical axis through the mass centre G is given by the angle θ which is sensibly small.

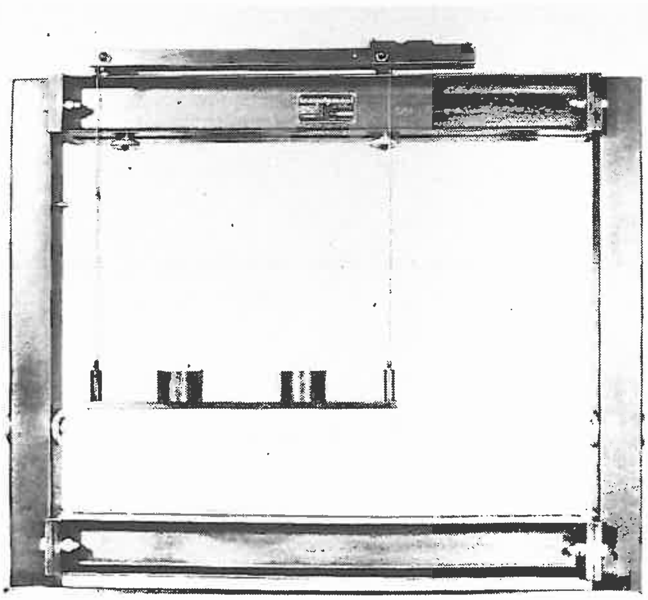


Fig 5.1

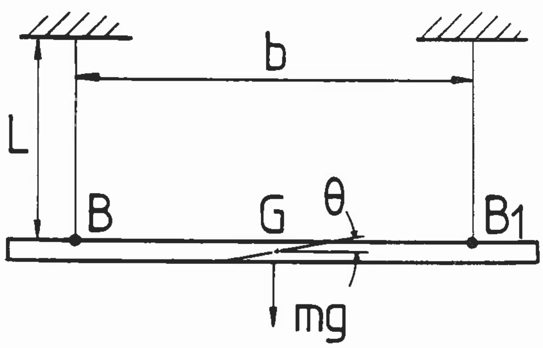


Fig 5.2

This equation of the angular motion is given by: $I\frac{d^2\theta}{dt^2} = -\frac{mgb^2}{4L}\theta$

which may be written $\ddot{\theta} + \frac{gb^2}{4k^2L}\theta = 0$

The motion is clearly simple harmonic and the period is given by

$$\tau = 4\pi\sqrt{\frac{k^2L}{gb}}$$

Note that 'I' is moment of inertia about swing axis through G. ($I = mk^2$)

Knowing the periodic time and the magnitudes of the various parameters, the radius of gyration k and hence the value of I are easily determinable.

2. Apparatus

The apparatus (shown in Fig 5.1) consists of a uniform rectangular bar B7 suspended by fine wires from the small chucks as used in Experiment 1. Thus the lengths of the suspension are easily altered by drawing the two wires through the chucks and tightening. The bar is drilled at regular intervals along its length so that two 1.85 kg masses may be pegged at varying points along it.

3. Method

The bar is suspended by the wires, the length L adjusted to a convenient size, and the distance between the wires 'b' is measured. The bar is the displaced angularly through a small angle. The time for 20 oscillations is then measured, from which the periodic time may be calculated.

The length of the wires L may then be adjusted, and a further 20 swings timed. The inertia of the body may be increased by placing two masses symmetrically on either side of the centre line distance x apart, and repeating the procedure for various values of L and the distance between the masses. The radius of gyration k of the sysstem may be calculated as previously outlined.

4. Results

These are best tabulated as shown in Fig 5.3

Test No	L (m)	x (m)	τ (s)	k (m)	k^2 (m^2)	m (kg)	$I = mk^2$ (kgm^2)
1.							
2.							
3.							
4.							

Fig 5.3 Table of Results for Bifilar Suspension

It is instructive to compare the value of 'I' obtained in a particular test with the value of I' determined analytically (using $\Sigma\delta m_{.}x^2$).

5. Discussion of Results

Some noteworthy points will have arisen as a result of your having performed this experiment. Write out your conclusions.

6. Further Considerations

1. How would one determine the radius of gyration, and hence moment of inertia, of a body using the bifilar suspension?

EXPERIMENT 6: MASS - SPRING SYSTEMS

1. Introduction

A helical spring, deflecting as a result of applied force, conforms to Hooke's Law (deflection proportional to deflecting force).

The graph of force against deflection is a straight line (see Fig 6.2).

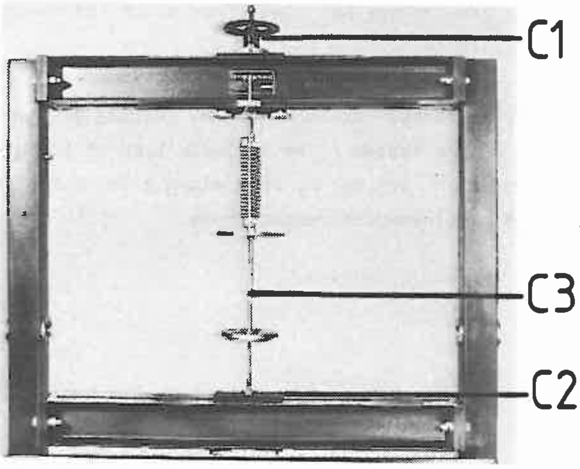


Fig 6.1.

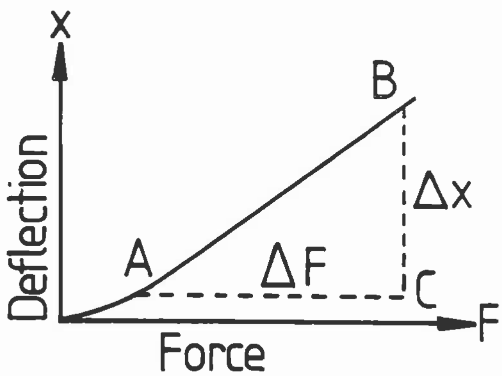


Fig 6.2.

Slope of the line $\frac{\Delta x}{\Delta F}$ is the "deflection coefficient" in metres per newton.

The reciprocal of this is commonly called the stiffness of the spring being the force required to produce unit deflection. A rigid body of mass M under elastic restraint, supported by spring (s), forms the basis of all analysis of vibrations in mechanical systems.

The basic equation is of the form:

$$M\ddot{x} = -kx$$

where k = stiffness of the spring

The motion is clearly simple harmonic of periodic time $\tau = 2\pi\sqrt{\frac{M}{k}}$

2. Apparatus

The required set-up for the experiment is shown in Fig 6.1. This utilises any one of three helical springs of varying dimensions which may be suspended from the upper adjustable assembly (C1) clamped to the top member of the portal frame.

To the lower end of the spring is bolted a rod and integral platform (C3) onto which bodies of mass 0.4 kg may be added. The rod passes through a brass guide bush, fixed to an adjustable plate (C2) attached to the lower member. A vernier depth gauge is supplied which, when fitted to the upper assembly with its movable stem resting on the top plate of the guide rod, can be used to measure deflection, and thereby stiffness, of a given spring.

3. Experimental Procedure

The springs to be tested are fixed to the portal frame, with the loading platform suspended beneath, and with the guide rod passing through the guide bush. The system is carefully adjusted to ensure that the guide bush is directly below the top anchorage point of the spring, since any misalignment incur errors in the experiments because of the guide rod rubbing against the guide bush. (To minimise friction at the bush it is advisable to smear a little grease or oil around the bush).

The length of the spring is then measured by the vernier gauge with the platform unloaded and then after each increment of weight is applied. Loads are applied until some suitable maximum load is reached, after which each load increment is removed and the extension on unloading is compared with that with the load increasing, resulting in a mean value of extension for the spring.

This should result in a straight line graph for extension plotted against load, from which the spring stiffness k may be determined.

The second part of the experiment involves adding varying masses to the platform, pulling down on the platform and releasing, - thus giving rise to vertical vibrations of the system. The periodic time of these vibrations is found by timing 20 oscillations. A graph is then drawn of τ^2 against M, from which g and m are found from the slope of the graph and the intercept of the line on the M axis produced respectively.

Note that $\tau^2 = \left(\frac{4\,\pi^2}{k}\right) M$

The mass of the rod and platform have to be included in the 'M' above. Enter the results in suitable tables and record the relevant data in respect of the spring(s) used (diameter of wire, number of coils, etc)

4. Results

Tabulate the results in respect of force and deflection for one of the springs in the first part of the experiment; the form of the table is shown in Fig 6.3. The corresponding graph, Fig 6.4, will give the value of k. Record data in respect of the spring as follow:

Mean coil diameter =

Mean wire diameter =

Enter the results of the second part of the experiments in a table as shown in Fig 6.5. The mass of the platform and guide rod assembly is included in the value for M. Plot a graph of the form shown in Fig 6.6. Draw the best straight line through the points and determine its slope.

From the intercept of the line on the 'M' axis, the effective mass of the spring is found (m). Compare the value of m thus obtained with the generally accepted value, viz. 1/3 mass of spring. The procedure can be repeated with the other springs provided.

5. Discussion of Results

State conclusions in the light of the results obtained. Basic theory verified?

6. Further Considerations

1. From the experiments so far performed, discuss the relative merits of each in calculating an accurate value for 'g'. Criteria for your comments should be:
   (a) Ease of experimentation
   (b) Inherent Inaccuracies
   (c) Ease of computation

2. Choosing some typical results, what error is introduced in calculating g by neglecting the effective mass of the spring?

M (kg)	Deflection x loading	Deflection x unloading	Mean x (mm)
0			
0.4			
0.8			
1.2			
1.6			
2.0			
2.4			
2.8			
3.2			
3.6			
4.0			

Fig 6.3.

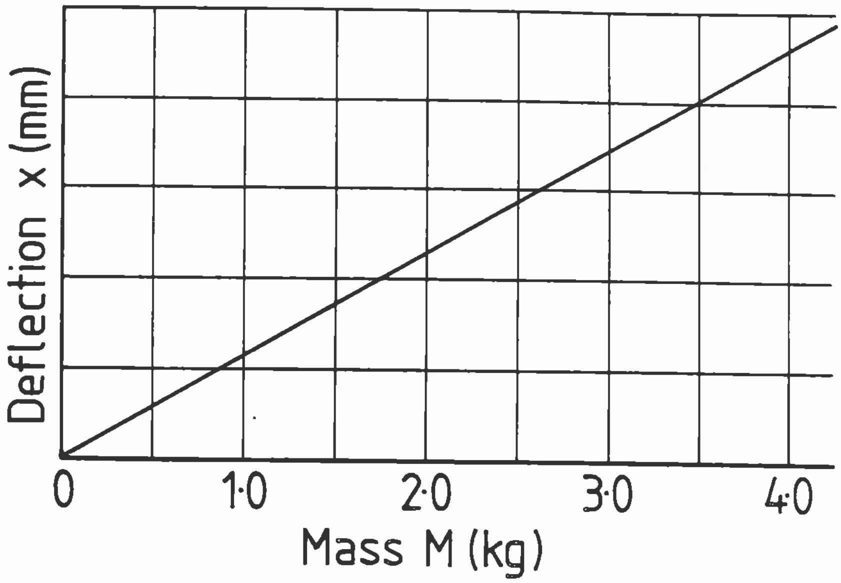


Fig 6.4.

M (kg)	Time for 20 oscillations	Period (s)	τ^2
1.47			
3.47			
3.87			
4.27			
4.67			
5.07			
5.47			

Fig 6.5

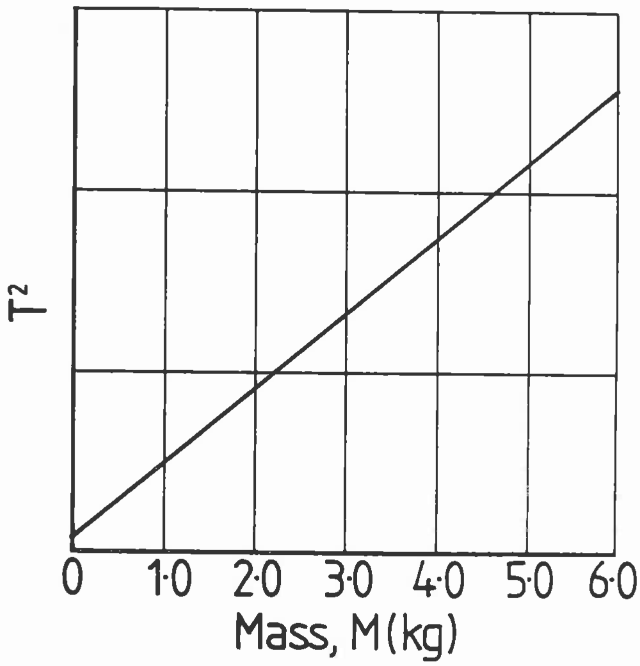


Fig 6.6.

EXPERIMENT 7: TORSIONAL OSCILLATIONS OF A SINGLE ROTOR

1. Introduction

This is an example of simple harmonic angular motion, the system comprising a rigid rotor at one end of an elastic shaft. It is called torsional vibration because of the twisting action on the shaft.

Analysis of this situation is analogous to the previous one (Experiment 6). Deflection x is replaced by θ, k, which was stiffness, is now torsional stiffness of the shaft, and mass M is replaced by I, the polar moment of inertia of the rotor.

The equation of motion is $I\ddot{\theta} = -k\theta$ clearly S.H.M.

It can be shown that Period, $\tau = 2\pi\sqrt{\frac{IL}{GJ}}$

where L is the effective length of the shaft
G is the modulus of rigidity of the material of the shaft
J is the polar second moment of area of the shaaft section

2. Apparatus

For experiments on undamped torsional vibrations, the inertia is provided by two heavy rotors, cylindrical in shape, one 168 mm diameter the other 254 mm diameter. The smaller diameter one H2 is depicted in Fig 7.1. The rotor is mounted on a short axle which can be fitted in either of the vertical members of the portal frame and secured by a knurled knob.

The rotor is fitted with a chuck designed to accept shafts of different diameter. The shaft is rigidly clamped by an identical chuck, which is an integral part of a bracket (I1) which is at the same height as the flywheel chuck and adjustable, relative to the base of the portal frame. Three steel test shafts are supplied with the rig, 3.18, 4.76 and 6.35 mm in diameter, each 965 mm long.

The inertia of the smaller rotor can be increased by bolting two pairs of steel arms to each side and attaching heavy masses at each end. Two pairs off masses are available of approx. 1800 g and 3200 g.

EXPPERIMENT 7(i) TO DETERMINE THE MOMENT OF INERTIA OF A FLYWHEEL

3. Procedure

The moment of inertia of a flywheel (one of the rotors would be most suitable), can be found experimentally by the falling weight method. The flywheel is mounted as described above so that it can rotate freely on an axle fitted to one of the vertical members of the frame.

(In the case of the smaller rotor with the added masses in position, it is necessary to clamp the rotor in the reversed position, since a complete revolution of the whole assembly is impossible with the rotor clamped inside the portal frame).

A suitable body of mass m is attached to a length of string which is wound around the circumference of the rotor and looped around a steel peg projecting from the rim. The body is allowed to fall through a measured height h to the ground and the time of descent t_1 recorded by a stop watch. (not supplied)

The number of revolutions n_1 of the wheel during this period of acceleration is found, also the number of revolutions n_2 and the corresponding time t_2 from the instant the body strikes the ground to the instant the rotor comes to rest. The length of string should be suitably adjusted so that the string detaches itself as the body strikes the ground. More than one test should be performed so as to obtain average values for n_1, n_2 and the times t_1 and t_2.

4. Theory

Apply the basic energy equation: $W = \Delta E$ to the two phases of the motion of the system.

Acceleration period

$$-T_f\, n_1\, (2\pi) = \tfrac{1}{2} m \left[v^2 - 0\right] + mg\left[0 - h\right] + \tfrac{1}{2} I \left[w^2 - 0\right]$$

Deceleration period

$$-T_f\, n_2\, (2\pi) = \tfrac{1}{2} I \left[0 - w^2\right]$$

Eliminating T_f from the above two equations gives

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} Iw^2 \left[\frac{n_1 + n_2}{n_2}\right]$$

From which 'I' can be calculated

Notation

m = mass of the falling body (kg)
h = height of fall (m)
v = maximum velocity of the body striking ground (m/s)
w = corresponding maximum angular velocity of the wheel (rad/s)
T_f = Frictional torque at the bearing of the wheel (Nm)
n_1 = number of revolutions of the flywheel required to wind up the string from the ground to the starting point
n_2 = number of revolutions the wheel makes after the falling body strikes the ground (deceleration period)

$$n_2 = \frac{w\ t_2}{4\pi}$$

and $$h = \frac{0 + v}{2} . t_1 \quad (t_1 = \text{time of fall of the body (s)})$$

$$\therefore \quad v = \frac{2h}{t_1} \quad \text{and} \quad w = \frac{v}{r} \qquad r = \text{effective radius (m)}$$

t_2 = time for wheel to come to rest after the falling body has reached the ground (s).

When performing a practical test the value of m should not be too big, otherwise the time of the second phase of the motion runs into many minutes. A value m equal to about 0.05 kg is suggested.

'I' comes to about 0.18 kg m ^{2}.

EXPERIMENT 7 (ii): FREQUENCY OF TORSIONAL OSCILLATIONS (SINGLE ROTOR SYSTEM)

Having determined a value for I for a particular rotar by the method described in Experiment 7 (i) using one of the three shafts, the frequency of torsional oscillations of a single rotor system can be found experimentally and the result compared with theoretical prediction.

5. Procedure

The shaft is passed through the central hole in the bracket, so that it enters the chuck on the flywheel and the latter is then tightened. The bracket is then moved along the slotted base until the distance between the jaws of the chuck corresponds to the required length L. The chuck on the bracket is then tightened.

Having ensured that the shaft is securely gripped in the jaws, the rotor (flywheel) may be displaced angularly and the time for 20 oscillations is recorded. The distance between the chucks is then varied in suitable increments by sliding the bracket, and values of periodic time corresponding to various lengths of shaft tabulated.

6. Results

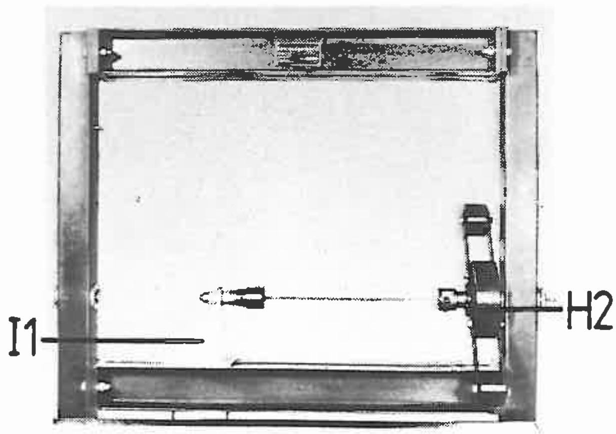


Fig 7.1

L (mm)	Time for 20 oscillations	Period τ (s)	τ^2
100			
150			
200			
250			
300			
375			
450			

Fig 7.2

Record the results in a table of the form of 7.2. Plot a graph of τ^2 against L as shown in Fig 7.3.

From the slope of the graph $\frac{4\pi^2 . I}{GJ}$ determine the value of G and compare with the generally accepted value.

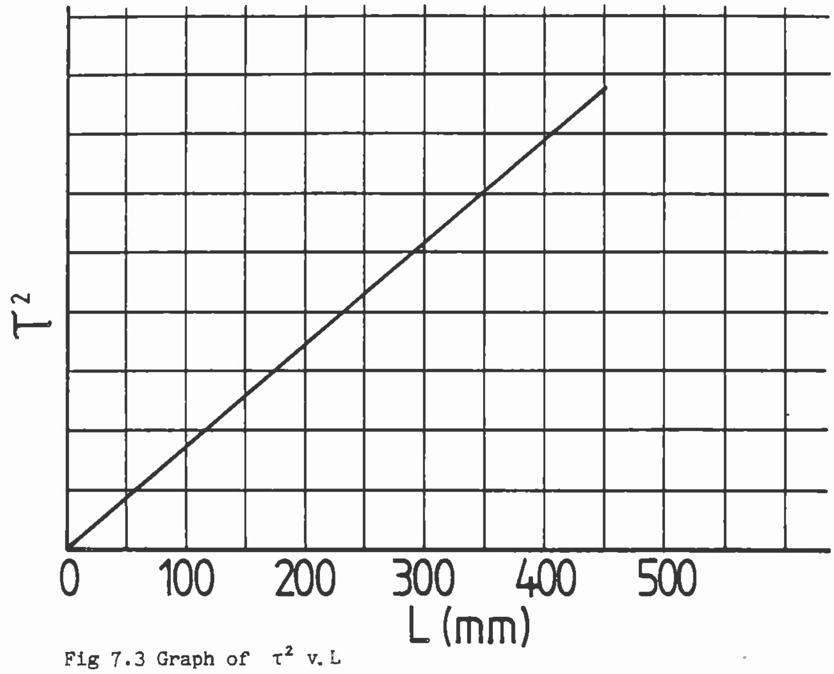


Fig 7.3 Graph of τ^2 v. L

7. Further considerations

1. Using the falling weight method how would the results be affected if the string were wrapped around the axle of the wheel instead of around its rim?
2. What change(s) in procedure would be necessary if a stepped shaft were used instead of one of uniform section throughout its length.

EXPERIMENT 8: TORSIONAL OSCILLATIONS OF A SINGLE ROTOR WITH VISCOUS DAMPING

1. Introduction

In this experiment, the effect of including a damper in a system undergoing torsional oscillations is investigated. The amount of damping in the system depends on the extent to which the conical portion of a rotor is exposed to the viscous effects of a given oil.

2. Theory

The equation of the angular motion is given by

$$\frac{I\, d^2\theta}{dt^2} = -C \frac{d\theta}{dt} - k\theta$$

which may be written

$$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0$$

where $a = \frac{c}{I}$ and $b = \frac{k}{I}$

The solution for angular displacement is given by

$$\theta = C.e^{*}.\cos(pt + \psi)$$

where $\Phi = \sqrt{b - \frac{a^2}{4}}$ N.B., $* = -\frac{a}{2}t$ and $\# = \frac{a}{2}(n\tau)$

and C and ψ are constants

Periodic time $\tau = \frac{2\pi}{p}$

Fig 8.1

Measuring amplitudes on the same side of the near position, the nth oscillation is given by $\frac{x_o}{x_n} = e^{\#}$

n is a positive integer corresponding to the number of complete oscillations starting at a convenient datum (t = 0)

Putting n = 1 gives the logarithmic decrement $\log_e\left[\frac{x_n}{x_{n+1}}\right] = \frac{a}{2}\tau$. This is all that is needed by way of basic theory.

3. Apparatus

The apparatus for this experiment consists of a vertical shaft gripped at its upper end by a chuck attached to a bracket (k_1) and at its lower end by a similar chuck attached to a heavy rotor (K3).

The rotor (K3) is suspended over a transparent cylindrical container (K4) containing damping oil. The oil container can be raised or lowered by means of knurled knobs on its underside, thereby altering the area of contact between the oil in the container and the conical portion of the rotor. This effectively varies the damping torque on the rotor when the latter oscillates. A trace of the damped oscillations can be recorded on paper wrapped round a drum mounted above the flywheel. Unit K2 consists of a pen-holder and pen which can be adjusted so as to make proper contact with the paper; the unit undergoes a controlled descent over the length of the drum by means of an oil dashpot clamped to the main frame.

Shafts of various diameters may be used, but due to the location and necessary fine adjustment of the oil container, the length is restricted to about 0.75 m. The angular displacement of the flywheel may be measured by means of a graduated scale on the upper rim of the rotor. An etched marking on the frame serves as a datum for the measurement of angular displacement.

4. EXPERIMENT 8 (i) DETERMINATION OF DAMPING COEFFICIENT

The cylindrical container (K4) is first filled with oil to within about 10 mm of the top and the knobs underneath adjusted so that the oil surface is level with one of the upper graduations on the conical portion of the rotor. d = 175 mm giving maximum damping is suggested (Details of the graduations are given in Fig 8.3)

Having selected and fitted a suitable shaft, the length of the shaft between the two inside faces of the chuck, together with the diameter of the shaft, should be noted. The rate of descent of the pen in mm/second should be measured by timing the descent of the pen over a fixed length of paper, using a stopwatch. (Not supplied)

The torsional oscillations of the system for the chosen damping condition can then be recorded. The pen is then raised to the top of the paper on the drum and the rotor through a suitable angle (about 40 deg) and released. A trace of the oscillations can then be obtained by bringing the pen into contact with the paper (using the thumb-nut on the support) and allowing the pen to descend.

A trace of the amplitude of oscillation will be recorded showing decay of the vibration, due to the damping. The rate of descent of the pen providing a time scale.

From the given trace, Fig 8.1, measure five successive amplitudes starting with the initial one (n = 0) and tabulate results as in fig 8.3.

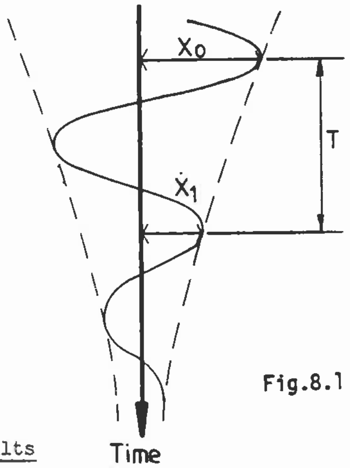


Fig.8.1

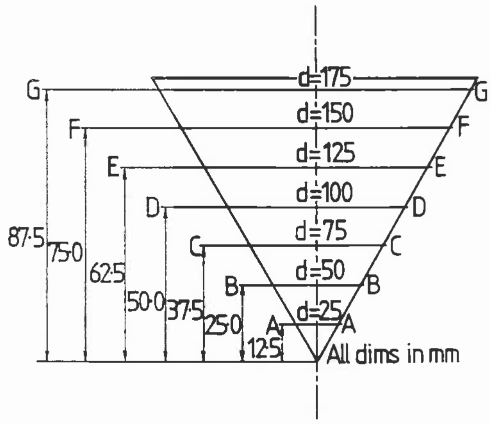


Fig 8.2.

5. Results

n	x_n (mm)	$\log_e \frac{x_o}{x_n}$
0	x_0 =	0
1		
2		
3		
4		
5		

Plot a graph of $\log_e \frac{x_o}{x_n}$ to a base of n.

A straight line through the origin confirms that the damping is viscous.

Slope of the line is equal to $\frac{a\tau}{2}$ (the logarithmic decrement).

The period can be found by timing a convenient number of oscillations using a stop watch, whereupon the constant "a" is determined and hence the value of the damping coefficient (the torque per unit angular velocity) in Nm/rad s^{-1}. The polar moment of inertia of the rotor is determined as in Experiment 7(ii).

6. EXPERIMENT 8 (ii) INVESTIGATION OF HOW THE DAMPING COEFFICIENT DEPENDS ON THE DEPTH OF IMMERSION OF THE ROTOR IN THE OIL

Repeat experiment 8(i) for each oil level as defined by the seven graduations on the conical portion of the rotor.

The damping coefficient depends on the area 'A' of the curved surface of the conical portion of the rotor exposed to viscous damping. This area is equal to πrl where r is the radius of base of core and l is the slant height equal to $\sqrt{r^2 + h^2}$.

Plot a graph of damping coefficient to a base of 'A' times mean radius.

7. Results

These are best tabulated as shown in Fig 8.3..

r (mm)	Mean radius r_m (mm)	Area A (mm^2)	A. r_m (mm^3)	Period τ (s)	Constant 'a'	Damping Coeff. (Nm/rad s^{-1})
12.5	6.25					
25	12.5					
37.5	18.75					
50	25					
62.5	31.25					
75	37.5					
87.5	43.75					

Fig 8.3 Results of Torsional Oscillation with Viscous Damping

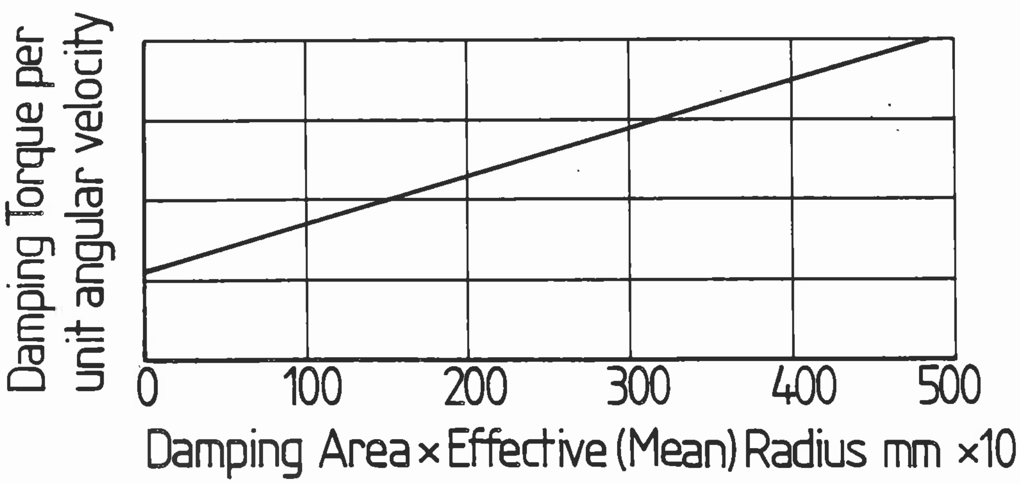


Fig 8.4

State the probable relationship between the two parameters.

EXPERIMENT 9: TORSIONAL OSCILLATIONS OF TWO ROTOR SYSTEM

1. Introduction

With the addition of a second rotor, the apparatus described in Experiment 7(ii) can be used to investigate the oscillation of a two rotor system. For such a system the periodic time is given by:-

$$\tau = 2\pi\sqrt{\frac{I_1 I_2 L}{GJ\,(I_1 + I_2)}}$$

where I_1 and I_2 are the moments of inertia of the two rotors
L is the length of the shaft between the rotors
G is the modulus of rigidity of the material of the shaft
and J is the polar second moment of area of the shaft section.

2. Apparatus

The apparatus (see Fig 9.1) used is that of Experiment 7, with the bracket (I1) replaced by a second rotor (H1) which is free to rotate on a axle fixed to the left-hand vertical member of portal frame. Chucks are fitted to both rotors so that shafts of various diameters can be used. Since the axles of both rotors are fixed to their respective vertical members, the length of the shaft may not be varied in this case but three shafts of different diameter are supplied and three combinations of different inertias are possible.

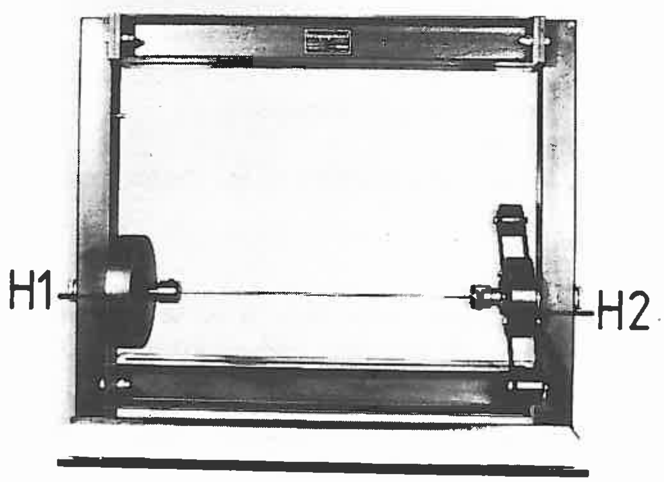


Fig 9.1

3. Experimental Procedure

One of the shafts is clamped between the two rotors H_1 and H_2 of predetermined inertia. The effective length of the shaft measure between the jaws of the chucks is then recorded. The chucks must be carefully tightened to ensure that neither rotor can slip relative to the shaft. Each rotor is rotated through a small angle in opposite directions and then released. Torsional oscillations of the system are thereby set up and the time for 20 oscillations recorded.

Hence the periodic time of the system may be determined and compared with the theoretical value given by the formula quoted in the introduction. The moments of inertia of the rotors should be determined by the method described in Experiment 7.

4. Results

The results for various rotor and shaft systems are recorded in a table of the form shown in Fig 9.2

Polar second moment of area $J = \frac{\pi d^4}{32}$

The generally accepted value of G for steel is 82 GPa and for g 9.81 m/s².

Shaft diam mm	I_1 kg m²	I_2 kg m²	Time for 20 oscillations	period τ	Theoretical value of period
3.17					
3.17					
4.76					
6.35					

Fig.9.2.

5. Further Considerations

When oscillating torsionally the two rotors oscillate back-to-back about a non moving section of the shaft, called the "node". It is instructive to locate the position of the node for a given pair of inertias and their shaft; one can do this by introducing a third (dummy) rotor in the form of a cardboard disc (negligible inertia) and moving it along the shaft to a position where it becomes fixed in space.

EXPERIMENT 10: TRANSVERSE VIBRATION OF A BEAM WITH ONE OR MORE BODIES ATTACHED

1. Introduction

The frequency of transverse vibrations of a beam with bodies attached is identical to the critical (whirling) speed of a shaft, of the same stiffness as the beam, carrying rotors the masses of which correspond to those of the bodies on the beam. One has to think in terms of small size rotors, otherwise gyroscope effects are involved. In the case of a beam with just one body attached, the basic theory is the same as that dealt with in Experiment 6. For a beam with two or more bodies attached, other methods are used to determine frequency of free transverse vibrations. Examples are as follows:

(i) Rayleigh or energy method; (gives good result)
(ii) Dunkerley equation (only approximate, but quite adequate)
(iii) Rigorous (accurate) analysis (arduous)
(iv) Experimental analysis, using the equipment described below, (fairly simple and quick)

2. Apparatus

The basic apparatus for this experiment is shown in Fig 10.1. A bar of steel of rectangular cross-section (E6) is supported at each end by trunnion blocks. The left-hand support (D1) pivots in two ball bearings carried in a housing located on the inside face of the vertical frame member.

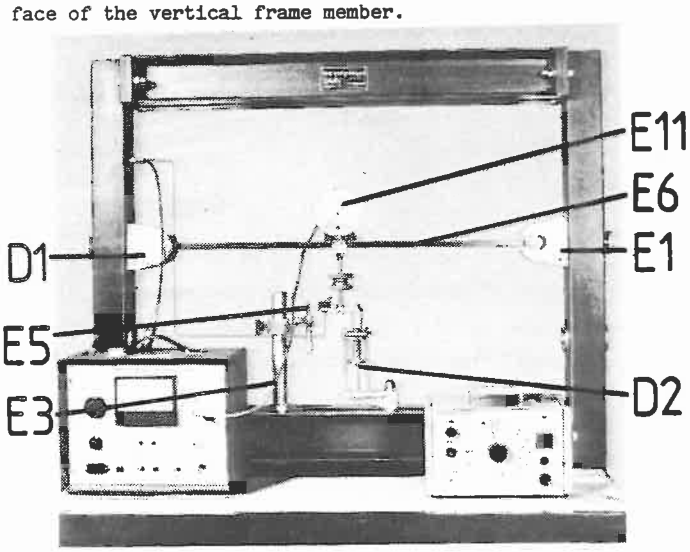


Fig 10.1

The right-hand support consists of two roller bearings which are free to move in a guide block, located on the inside face. At the centre of the beam is bolted a small motor carrying two "out-of-balance" discs (part of E11). The motor is connected via leads to the precision speed control unit, complementary to E11, which enables a wide range of exciting frequencies to be applied to the beam.

Clockwise rotation of the speed control knob on the speed control unit increases the speed of the motor - thus increasing the out-of-balance rotating force produced by the unbalanced discs. As the speed increases, as indicated by the speed meter on the control unit, the beam begins to vibrate transversely; over a discrete band of frequencies increasingly larger amplitudes of vibration are produced which reach a peak at a frequency corresponding to the frequency of free natural transverse vibration of the system, i.e. beam plus added components.

3. Experimental Procedure

Bodies of different size mass M are suspended below the motor. For each value of m the speed control is adjusted until the beam vibrates at its natural frequency.

In order to determine accurately the exact value on the speed meter, it is expedient to take the beam through the range of excessive amplitudes several times, noting the limits of the range and thus gradually locating the frequency at which the amplitude and resultant noise appears greatest.

Record observations in a suitable table; see Fig 10.2

m kg	Frequency f (hz)	f^2	$\frac{1}{f^2} \times 10^3$
4.8			
5.2			
6.8			
8.0			
9.2			
10.8			
11.6			
13.2			
14.4			

Fig 10.2 Table of Results for Expt 10(i)

4. Results

A graph of $\frac{1}{f^2}$ to a base of m gives a straight line. (see Fig 10.3)

The intercept on the vertical axis is equal to $\frac{1}{f_b^2}$

f = natural frequency of the system, i.e. beam plus added components

f_b = natural frequency of the beam by itself.

Dunkerley's equation is applicable to this situation, it is given by:

$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_b^2}$$

Here f_1 = natural frequency of a corresponding light beam with mass m attached.

Clearly when $m = 0$, $f_1 = \infty$ and $f = f_b$

Evaluate f_b and compare with the theoretical value obtained from:

$$f_b = \frac{\pi^2}{2}\sqrt{\frac{EI}{m_o L^3}}$$

where L = Length of the beam (m)

E = Modulus of elasticity of material of the beam (N/m^2)

I = Second moment of area of the beam section (m^4)

m_o = Mass of the beam by itself (kg) (no mass(es) attached)

Also, from the graph, when the system is not vibrating (period $\tau = 0$) $f = \infty$ and $\frac{1}{T^2} = 0$.
The corresponding value of mass m is then equal to m_e, the equivalent mass of the beam.
$m_e = \lambda . m_o$ (λ = some constant). Determine the value of λ. How does it compare with the generally accepted value of 0.5?

5. Further Considerations

The validity of the Dunkerley equation in the more familiar form can be tested by moving the motor with out-of-balance discs away from the centre of the beam and attaching a heavy body of known mass at some other arbitrary point on the beam. The Dunkerley equation then becomes

$$\frac{1}{f^2} = \frac{1}{f_T^2} + \frac{1}{f_2^2} + \frac{1}{f_b^2}$$

The f_2 in this equation could be the variable parameter and a graph plotted similar to the one described above. Certain extra components (not supplied with the standard equipment) would be required to perform this additional test. They are: a special block for attaching extra masses to the beam, and a suitable vibration generator of variable frequency.

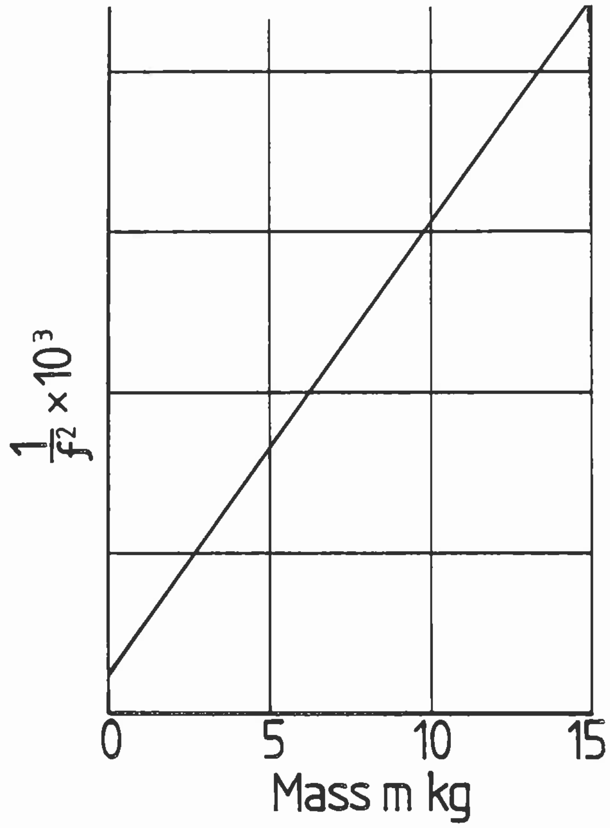


Fig 10.3

graph of $\frac{1}{f^2}$ against m for the system

EXPERIMENT 10(i) DAMPED TRANSVERSE VIBRATION OF A BEAM

6.1 Introduction

Damping forces are counteracting forces in a vibration system which gradually reduce the motion. Damping occurs in all natural vibrations and may be caused by Coulomb friction (rubbing between one solid and another), or viscous resistance of a fluid as in this experiment on damped transverse vibration of a beam where a dashpot is used.

7.2 Apparatus

This is shown in Fig 10.1 (i.e. the same set up as for Experiment 10(i), but with certain additions). In this experiment, amplitude of vibration and phase angle are required. A dashpot (D2) and for support for the latter, (E2) are provided. The amplitude and phase angle are determined very accurately at any exicting frequency by the use of a contactor (E5) and micrometer mounted vertically on its underside. The electric circuit, of which a stroboscope is a part, is completed when the contact element E5 touches the plunger of the micrometer.

8. Experimental Procedure

Allow the speed control unit time to warm-up, then adjust the micrometer plunger so that it just touches the contactor; with the stroboscope switched to EXTERNAL stimulus, a discharge occurs on contact, the micrometer reading should be taken, and this value is then used as a datum position from which values of amplitude may be determined.

The motor is then energised, producing a definite amplitude at a predetermined frequency. To determine the amplitude the micrometer head will now need to be lowered and then brought up again to produce contact. It is important that the strobscope discharges at a uniform frequency, and so careful adjustment must be made to ensure steady conditions. At this point the amplitude of the vibration may be found by comparing the new micrometer reading with that of the original datum position.

The phase angle may also be found by focussing the stroboscope on the graduated disc on the motor shaft; since the stroboscopic discharge should be at a frequency corresponding to the rotational speed of the motor, the disc may be effectively stopped and the phase angle corresponding to the datum mark on the motor read off. By following this procedure for a range of frequencies, the effect of damping may be assessed by varying the piston area of the dashpot and thus altering the damping characteristics of the system.

The two orifice plates inside the dashpot can be rotated relative to one another thereby varying the effective area and the results obtained with these settings can be compared with an undamped condition (the system minus dashpot). Graphs are then plotted of amplitude and phase angle against the frequency ratio (w/w_n) i.e. (exciting frequency / natural frequency).

N.B. At low frequencies, phase angle may not be obtainable.

9. Results

Plot the results in the form of Fig 10.4 - 10.6

Tables of results (Fig 10.4 - 10.6) show the effect of increasing damping on amplitude and phase angle. For each damping condition a graph of amplitude against frequency can be plotted, from which a value for the natural frequency for each damping condition can be found.

Typical values obtained in this way are as follow:

No Damping	17.36 Hz
Light Damping	17.50 Hz
Heavy Damping	17.58 Hz

and from these values the frequency ratio can be found being the exciting frequency / natural frequency. Fig 10.7 and 10.8 are typical graphs of amplitude and phase angle plotted against frequency ratio.

Motor speed (rev/min)	$\frac{w}{w_n}$	Phase Angle log (deg)	Amplitude x_{max} (mm)
500			
600			
700			
800			
900			
980			
1000			
1010			
1020			
1040			
1050			
1055			
1060			
1075			
1100			
1200			
1300			
1400			
1500			
1800			
2000			
2500			

Fig 10.6 Table of results heavy damping

Motor speed (rev/min)	$\frac{w}{w_n}$	Phase Angle log (deg)	Amplitude x_{max} (mm)
500			
600			
700			
800			
900			
980			
1000			
1010			
1020			
1040			
1050			
1055			
1060			
1075			
1100			
1200			
1300			
1400			
1500			
1800			
2000			
2500			

Fig 10.5 Table of results light damping

Motor speed (rev/min)	$\frac{w}{W_2}$	Phase Angle log (deg)	Amplitude x_{max} (mm)
500			
600			
700			
800			
900			
980			
1000			
1010			
1020			
1040			
1050			
1055			
1060			
1075			
1100			
1200			
1300			
1400			
1500			
1800			
2000			
2500			

Fig 10.4 Table of results no damping

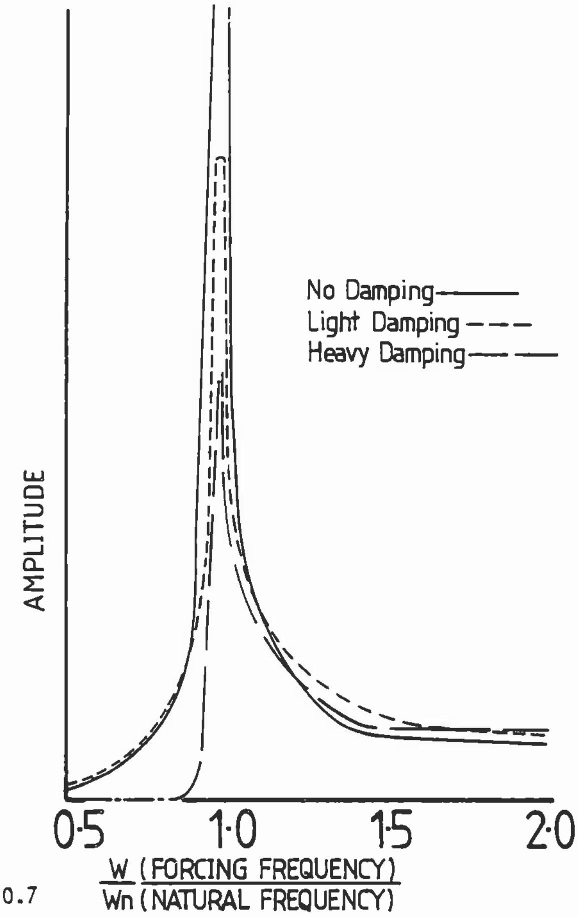


Fig.10.7

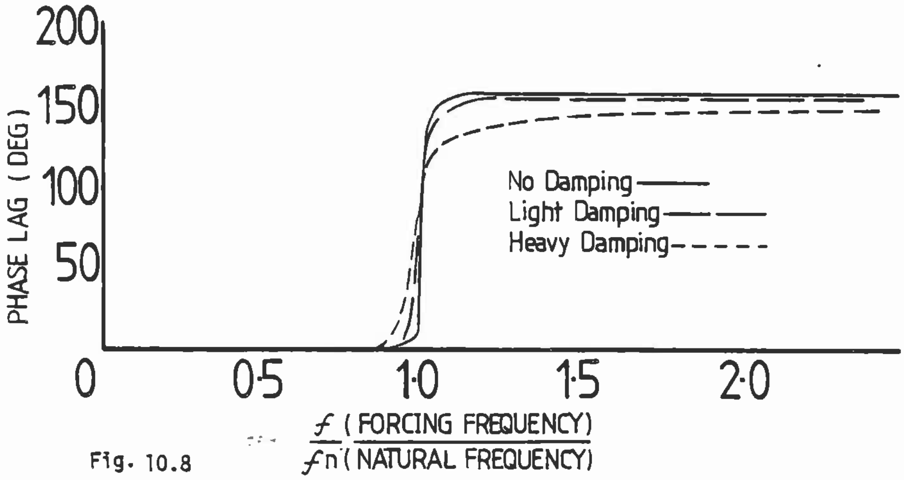


Fig. 10.8

EXPERIMENT 11: UNDAMPED VIBRATION ABSORBER

1. Introduction

Excessive vibrations in engineering systems are generally undesirable and therefore avoided for the sake of safety and comfort. It is possible to reduce untoward amplitudes by attaching to the main vibrating system an auxiliary oscillating system, which could be a simple mass-spring system or pendulum. In this experiment, the vibration absorbability of a double cantilever system is examined.

2. Apparatus

Fig 11.1 shows the vibration absorber clamped below the motor. It comprises two bodies of equal mass fixed equidistant from the midpoint of the horizontal cantilever. (The distance apart of the bodies is varied until the system is "tuned").

3. Procedure

For a given frequency, the masses of the vibration absorber is adjusted so that the energy of vibration is transmitted to the absorber so that the amplitude of the main (primary) system, i.e. the motor and beam, is reduced to zero.

The aim is to determine the length l, the distance of the centre of each of the bodies from the midpoint of the cantilever, so that the natural frequency of transverse vibration of sub system corresponds to the running speed of the main (primary) system, i.e. the motor and the beam.

The formula for determining 'l' is

$$f = \frac{1}{2\pi}\sqrt{\frac{3EI}{ml^3}}$$

Here f = natural frequency of the sub (auxiliary) system
m = mass of each of the bodies
EI = flexural rigidity of the double cantilever

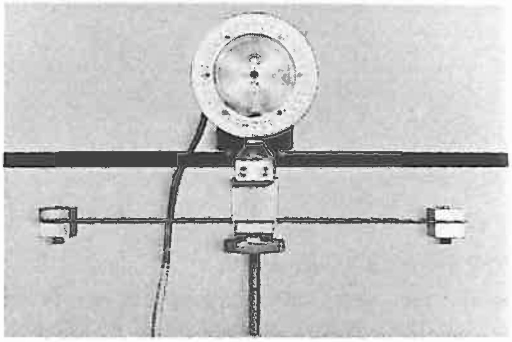


Fig 11.1

EXPERIMENT 12: FORCED VIBRATION OF A RIGID BODY - SPRING SYSTEM WITH NEGLIGIBLE DAMPING

1. Introduction

When external forces act on a system during its vibratory motion, it is termed forced vibration. Under conditions of forced vibration the system will tend to vibrate at its own natural frequency superimposed upon the frequency of the excitation force.

Friction and damping effects, though only slight are present in all vibrating systems; that portion of the total amplitude not sustained by the external force will gradually decay. After a short time, the system will vibrate at the frequency of the excitation force, regardless of the initial conditions or natural frequency of the system. In this experiment, the natural frequency of the forced vibration of a rectangular section beam is observed and coompared with the result determined analytically.

2. Theory

The system comprises:

(i) a beam AB, of length b, sensibly rigid, of mass m, freely pivoted at the left-hand end. See Fig 12.2
(ii) a spring of stiffness S attached to the beam at the point C
(iii) a motor with out-of-balance discs attached to the beam at D

M = mass of the motor including the two discs

The equation of the angular motion is given by

$$I_A \frac{d^2\theta}{dt^2} = (F_0 \sin wt)L_1 - (S L_2 \theta)L_2$$

$$I_A = M L^2 + \frac{mL^2}{3}$$ the M of I of the system about the pivot axis

θ = angular displacement of the beam

F_0 = maximum value of the disturbing force

w = angular velocity of rotation to the discs

The above equation reduces to the form

$$\frac{d^2\theta}{dt^2} + b_0\theta = A \sin wt$$

The values of the constants b, A and w are known. Only the steady-state motion is of interest

i.e. $\theta = \frac{A \sin wt}{b - w^2}$

Amplitude, $\theta \max = \left| \frac{A}{b - w^2} \right|$

Resonance occurs when $b - w^2 = 0$

So the critical angular velocity of the motor is given by $\sqrt{b}$.

Note that in practical circumstances the amplitude, although it may be very large, does not become infinite because of the small amount of damping which is always present.

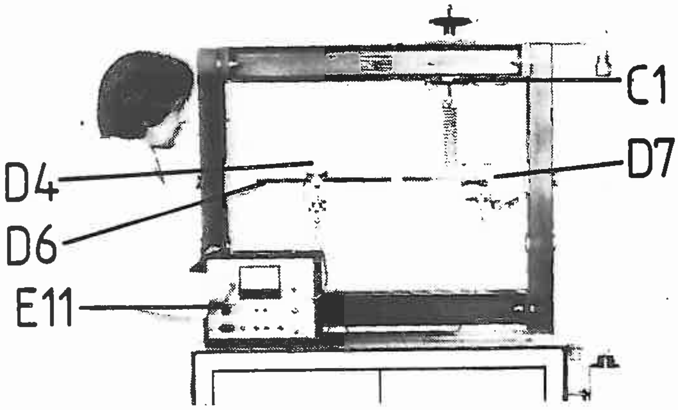


Fig 12.1.

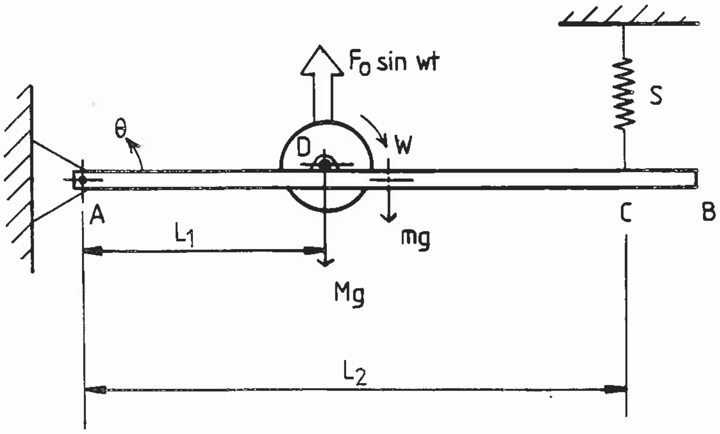


Fig 12. 2.

2. Apparatus

The apparatus as shown in Fig 12.1 consists of a rectangular beam (D6). supported at one end by a trunnion pivoted in ball bearings located in a fixed housing. The outer end of the beam is supported by a helical spring of known stiffness bolted to the bracket C1 fixed to the top member of the frame. This bracket enables fine adjustments of the spring to be made, thus raising and lowering the end of the beam.

The motor unit E11 (see Fig 12.3) is rigidly bolted to the beam with additional masses placed on the platform attached below. The forcing motion is provided by two out-of-balance discs on the output shaft of the belt driven unit (D4), the forcing frequency being adjusted by means of the speed control unit.

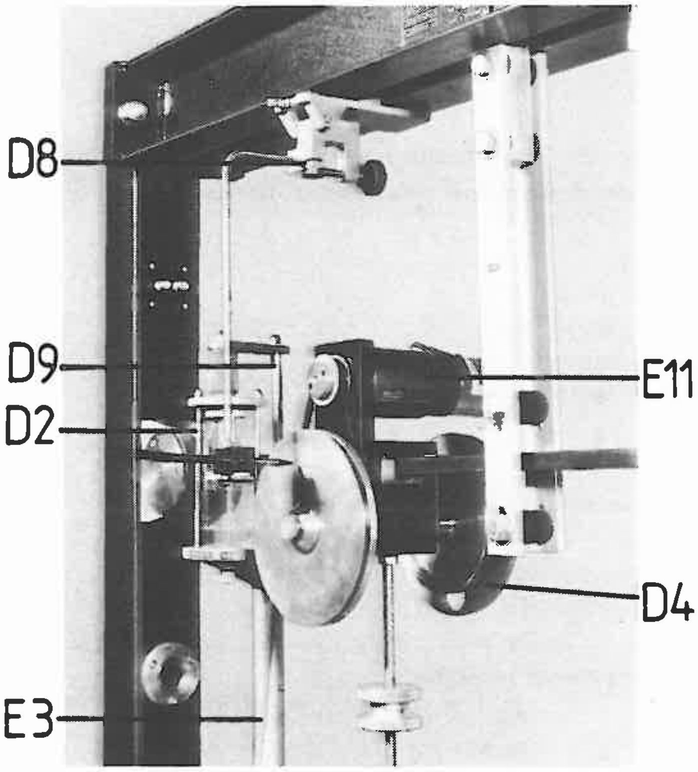


Fig 12.3

The chart recorder (D7), fitted to the right-hand vertical member of the frame provides the means of obtaining a trace of the vibration. The recorder unit consists of a slowly rotating drum driven by a synchronous motor, operated from auxiliary supply on the E11 speed control unit. A roll of recording paper is fitted adjacent to the drum and is wound round the drum so that the paper is driven at a constant speed. A felt-tipped pen is fitted to the free end of the beam; means are provided for the drum to be adjusted horizontally so that the pen just touches the paper. The paper is guided vertically downwards by a small attachable weight. By switching on the motor, a trace can be obtained showing the oscillations of the end of the beam.

If the amplitude of vibration near to the resonance condition is too big then extra damping can be introduced into the system by fitting the dashpot assembly (part numbers D2, D3 and D9) near to the pivoted end of the beam.

3. Experimental Procedure

The electrical lead from the synchronous motor is first of all plugged into the auxiliary socket on the control unit E11. The hand wheel of bracket C1 is then adjusted until the beam is horizontal and the chart recorder brought to a position where the pen just touches the recording paper.

The speed control unit can be switched on, resulting in forced vibration causing the beam to oscillate. It has been found that a frequency of about 2 hertz is suitable, the position of the motor being adjusted accordingly: the time for 20 oscillations will then be about 10 seconds. The chart recorder can be used to record the number of cycles performed by the beam in a given time (calculated, knowing the speed of the paper or, better still, by visual counting).

The pen is brought into contact with the paper, then a recording is made from which the number of cycles per unit time (i.e. the frequency) of the forced vibration beam can be calculated.

The speed of the paper on the chart recorder needs to be known: this is obtained by recording a trace for a convenient time, say 20 seconds, and then measuring the length of the trace. The speed in mm/s can then be calculated.

Determine the values of the relevant parameters as described in section 2 dealing with the theory: lengths L_1, L_2 magnitude of the masses m and M, also the stiffness of the spring.

4. Results and Calculations

4.1 Calculate: linear speed of paper on the drum the time of 'say' 20 vibrations, using a stop watch the time for one cycle (period of vibration) obtained two different ways the corresponding frequency.

4.2 Calculate relevant M of I.

Mass of motor with discs, M = kg

Mass of beam, m = kg

Lengths L_1 = m

L_2 = m

L = m

4.3 Calculate stiffness of the spring (as in Experiment No 6)

$$S = \frac{\text{deflecting force}}{\text{deflection}} = \quad \text{N/m}$$

4.4 Calculate frequency of the forced vibration

$$\text{The constant } b = \frac{S}{I_A} = \quad \frac{N\,m^{-1}}{kg\,m^2} \equiv s^{-2}$$

$$\therefore f = \frac{w}{2\pi} \text{ i.e. } \frac{b}{2\pi} = \quad \text{cycles/s (Hz)}$$ (Compare with the valves of f found in 4.1. above)

EXPERIMENT 13: FREE DAMPED VIBRATIONS OF A RIGID BOODY - SPRING SYSTEM

1. Introduction

During vibrations, energy is dissipated and thus a steady amplitude cannot be maintained without continuous replacement. Viscous damping in which force is proportional to the velocity affords the simplest mathematical treatment.

A convenient means of measuring the amount of damping present is to measure the rate of decay of oscillation. This is expressed by the term "logarithmic decrement", defined as the natural logarithm of the ratio of successive amplitudes on the same side of the mean position (see Fig 8.2)

In this experiment, the effect of the position of the dashpot and the corresponding damping coefficient are assessed in terms of the logarithmic decrement, measured by the decay in amplitude of a free vibration in a beam.

2. Theory

Referring to Fig 12.1,, the disturbing force F_0 sin wt is replaced by a damping force $cL_1 \frac{d\theta}{dt}$ downward.

The equation of the angular motion becomes $I_A \ddot{\theta} = -(c L_1 \dot{\theta})L_1 - (S L_2 \theta)L_2$

which can be put in the form:

$$\ddot{\theta} + a\dot{\theta} + b\theta = 0$$

The theory from now on is identical to that set out in Experiment 8 (the same symbols are used).

3. Apparatus

The apparatus used is that shown in Figs 12.2 and 12.3 in experiment 12, except that the exciter motor is not required since free vibrations only are of interest. The speed control box of E11 unit is required though in order to drive the drum on the recorder unit D7. The system is set vibrating freely by pulling down on the free end of the beam a short distance (15 - 25 mm) and releasing. The chart recorder is used to obtain a trace of just three successive amplitudes on the same side of the mean position. The damping is varied by moving the dashpot (D2) and its clamps along the beam, and also by the relative rotation of the two orifice plates in the dashpot to increase or decrease the effective area of the piston as in experiment 10 (ii).

4. Experimental Procedure

The speed control unit is switched on and the lead from the motor of recorder unit D7 is connected to the auxiliary supply socket on the control box E11. The dashpot is set at a particular distance L_1 (the distance from the trunnion mounting to the centre of the beam clamp (D9), and the beam is then pulled down a short distance, under the point of attachment of the spring, and released.

The recording pen is brought into contact with the paper to produce a trace of the decaying amplitude of vibration and thus a trace of the decaying applied amplitude is produced on the 'TELEDEDTOS' paper. For a given piston area, various values of L_1 can be selected and traces obtained. A different piston area is then chosen and the process repeated.

For each piston area and the value L_1, the trace is used to evaluate the logarithmic decrement; The periodic time of one complete oscillation, τ, is found in the manner described in section 3 of Experiment 12.

Recapitulating:

$$\text{Ln}\ \frac{x_0}{x_1} = \frac{a\tau}{2} \qquad \text{Constant 'a'} = \frac{cL_1^2}{I_A}$$

(Incidentally, constant 'b' $= \frac{SL_2^2}{I_A}$)

Hence the damping coefficient, c, the resisting force per unit relative velocity can be determined.

5. Results

It is suggested that the results are entered in two tables, see Fig 13.1, one relating to maximum damping (orifice plates in the dashpot set to give maximum area), the other relating to minimum damping.

Suggested format of each table:-

Length L_1 (m)	Ampl. ratio $\frac{x_0}{x_1}$	Log dec Log $\frac{x_0}{x_1}$	Period τ (s)	Constant 'a'	Damping Coeff c N/m s^{-1}
0.1					
0.15					
0.20					
0.25					

Fig 13.1

Plot, on the same graph, values of Damping Coefficient c against L^2. Typical plots are shown in Fig 13.2

Thus it can be shown that the logarithmic decrement - hence damping coefficient - varies according to the square of the distance from the dashpot. The position of the dashpot on the beam may be adjusted to produce any degree of damping, by consulting the graph. This information may be utilised in Experiment 14.

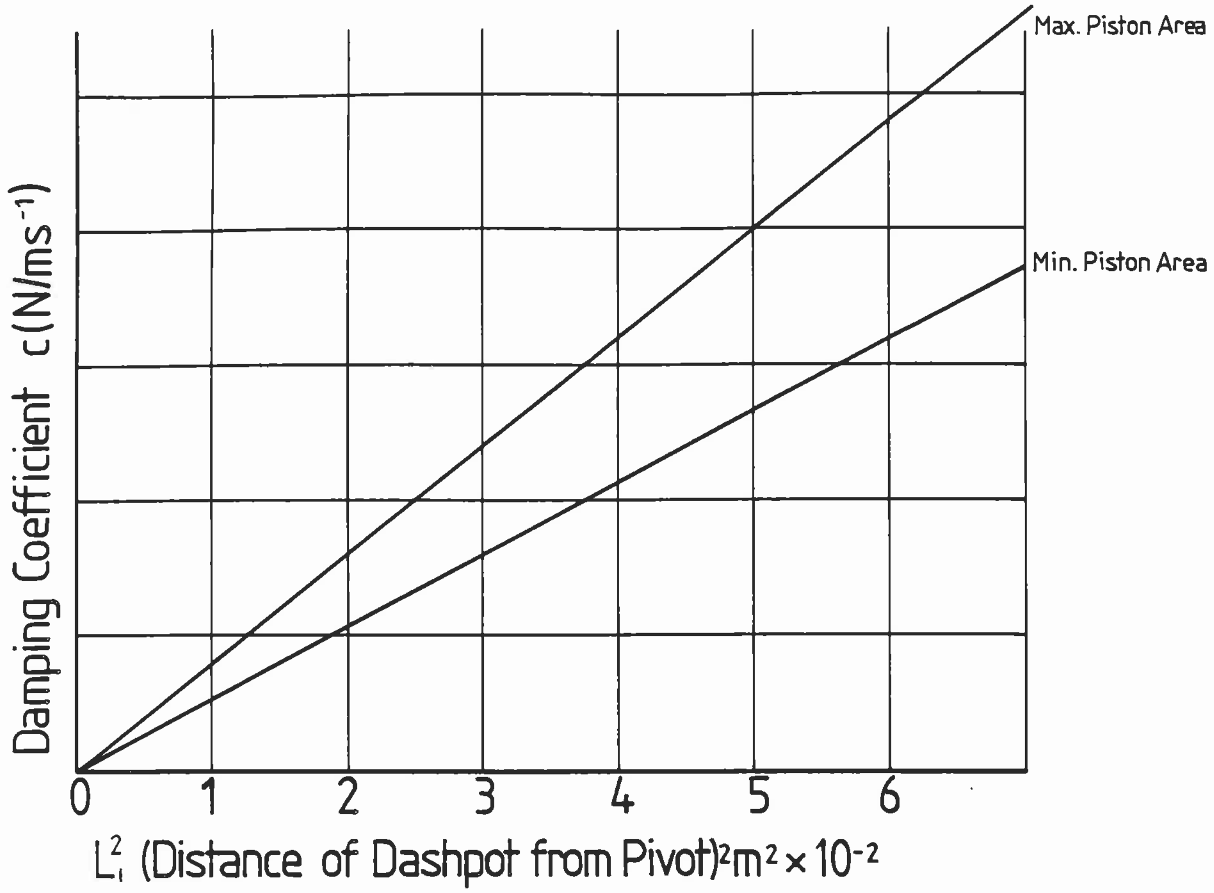


Fig 13.2 Graph of Damping Coefficient against L^2 for Damped Free Vibrations

EXPERIMENT 14: FORCED DAMPED VIBRATION OF A RIGID BODY- SPRING SYSTEM

1. Introduction

Having established the effect of viscous damping on free vibrations in the previous experimenet, the effect on forced vibration in now analysed. The means of assessing the relative magnitude of the forced vibration is to use the concept of "Dynamic magnifier" which is the ratio of the amplitude of the forced vibration to the deflection produced if the maximum value of the disturbing force 'F' were applied STATICALLY under the same elastic restraint.

2. Theory

The out-of-balance force F is given by $2mrw^2$ (two discs)

m = mass corresponding to hole in each disc (kg)

r = radius to centre of hole (m)

w = angular velocity of discs (rad/s)

Note that the ratio of the rotational speed of the discs to that of the motor is 22:72. This must be taken into account when calculating the angular velocity of the discs, w rad /s, from the speed indicated on the control unit.

Referring to Fig 12.1

The equation of the angular motion is given by

$$I_A \ddot{\theta} = (F \sin wt)L_1 - (c L \dot{\theta})L - (S L_2 \theta)L_2$$

which is reduced to the standard form:

$$\ddot{\theta} + a\dot{\theta} + b\theta = A \sin wt$$

Only the steady-state motion is of interest

$$\text{i.e. } \theta = \frac{A \sin(wt - \phi)}{\sqrt{(b - w^2)^2 + w^2a^2}}$$

$\emptyset$ = phase angle lag $\tan\emptyset = \frac{wa}{b - w^2}$

The other symbols have their usual meaning. Working in terms of angles the Dynamic Magnifier is given by

$$D_m = \frac{\theta\ max}{\theta_0}$$

where θ_0 = angular displacement of the beam due to the force F applied STATICALLY

and $\theta_0 = \frac{F\ L^1}{k}$ . . . 14.1

where $k = \frac{S L^2}{2}$ (the torsional stiffness of the beam)

Deflections measured are those of the end B of the beam and are given by $x = L\theta$

so $D_m = \frac{x_{max}}{L\ \theta^0}$ . . . 14.3

It can be shown that

$$D_m = \frac{1}{\sqrt{(1 - \frac{w^2}{b})^2 + \frac{w^2 a^2}{b^2}}}$$ . . . 14.4

And in nearly all practical circumstances, damping is "light", and therefore 'a' is sensibly small

Thus $D_m \doteq \frac{1}{1 - \frac{w^2}{w_n^2}}$ . . . 14.5

w = circular frequency of the forced vibration (rad/s)

w_n = circular frequency of free undamped vibration (rad/s)

3. Apparatus

The apparatus used is shown in Fig 12.1 with the dashpot added and the addition of one extra item, a plate clamped to the out-of-balance disc. The plate holds a piece of circular paper. See Fig 12.3

The recording pen if fitted to pivot (D8), which clamps to the upper member of the frame and may be clipped above the frame when not in use. The pen thus makes a trace of the locus of the point at any radius on the rotor, and - since the rotor is capable of vertical as well as rotational movement - a trace will be obtained from which the phase lag can be determined.

4. Experimental Procedure

The natural frequency of the system is first found as described in section 4 of Experiment 13, by analysising the free vibrations of the system, without the dashpot, from a trace produced on the chart recording unit D7.

The dashpot unit D2 is then fitted at a suitable point along the beam to give definite degree of damping (as determined in Experiment 13). The exciter discs are then rotated at a very low speed and a datum trace obtained on the paper mounted on the plate attached to the nearside disc. The position of the hole in the disc is then marked on the trace. The beam is then subjected to forced vibration by increasing the speed of rotation until a reasonable amplitude is obtained. A second "dynamic" trace is then obtained on the paper mounted on the plate. A trace should also be obtained on the chart recorder at the right-hand end of the beam (as in Experiment 12) in order to determine the amplitude of the vibrations. The procedure is repeated for different speeds below and above the critical speed to show how the value of dynamic magnifier varies with frequency for a given value of the damping coefficient.

Determine the phase lag from the traces recorded on the paper as shown in Fig 14.1. Note that the dynamic trace is displaced relative to the axis of rotation due to the vibration of the beam. If there were no phase lag betwen the exciting force and the resulting vibration, the dynamic trace would be displaced along the datum line corresponding to the out-of-balance force. By joining up the points of intersection of the two traces and drawing a line through the axis of rotation at right angles, the phase lag $\emptyset$ is determined for the various speeds of rotation of the discs.

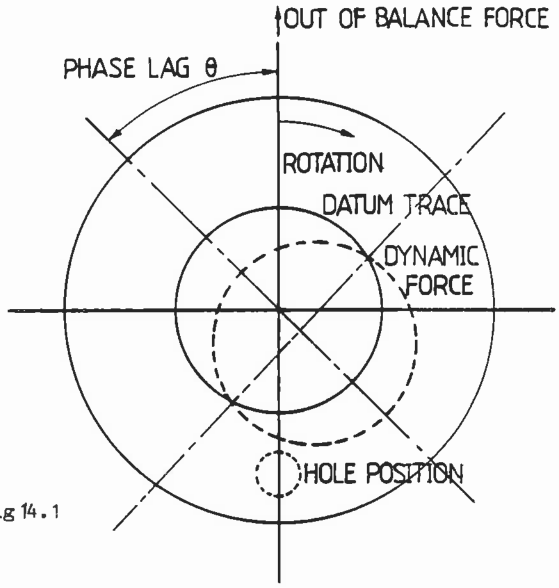


Fig 14.1

5. Results

The results are best tabulated as shown in Fig 14.2 the columns being number for the purpose of this explanation only.

There will be two tables, one for the case of no damping, the other for a definite degree of damping. Present a specimen set of calculations in respect of each table.

(i) Exciter Motor Speed (rev/min)	(ii) Angular Velocity of disc w (rad/s)	(iii) $\frac{w}{w_n}$	(iv) Amplitude x_{max} (mm)	(v) Phase Angle lag (deg)	(vi) "Static" Deflection (mm)	(vii) Dynamic Magnifier D_m
500						
550						
600						
625						
640						
650						
660						
675						
700						
800						
900						

Fig 14.2

Column (ii) $w = \left(\frac{N}{60} \times 2\pi\right) \frac{22}{72}$

Hence the ratio, column (iii)

Amplitude, column (iv), is obtained from the trace on the drum recorder D7, and the corresponding phase angle lag, column (v) is obtained in the manner already described. The "Static Deflection", column (vi), is obtained using equations 14.1 and 14.2.
Hence D_m using equation 14.3

Plot graphs of Dynamic Magnifier D_m and Phase Angle each to a base of the ratio $\frac{w}{w_n}$.

Typical graphs are shown in Figs 14.3 and 14.4. Similar results can be obtained for different degrees of damping.

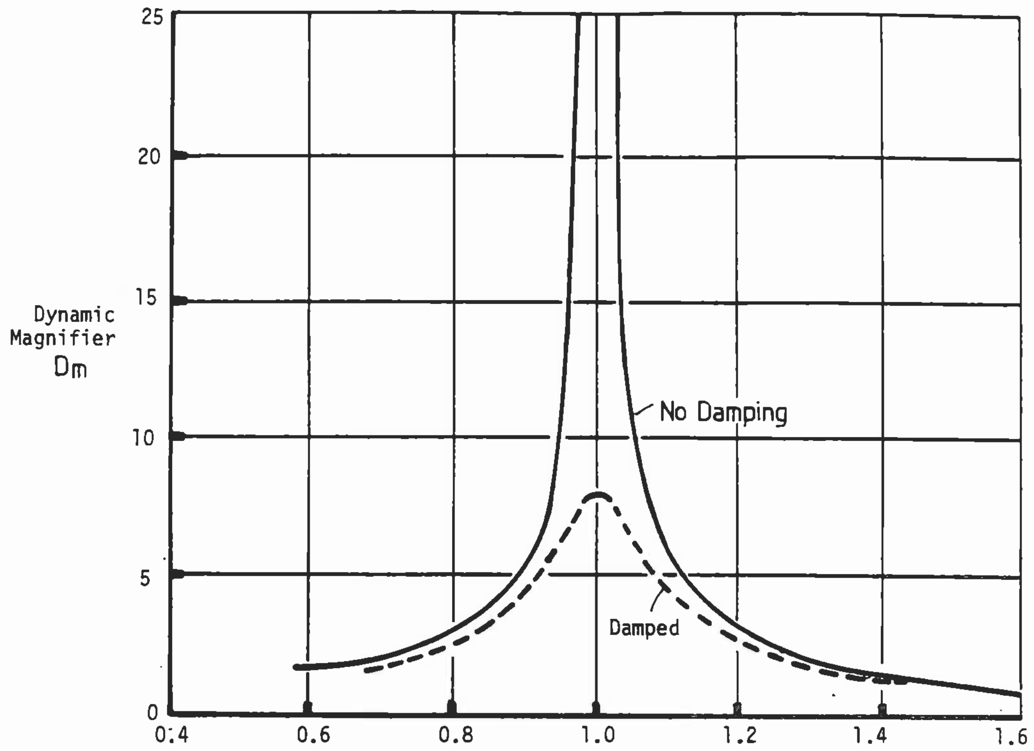


Fig. 14.3 Dynamic Magnifier against ratio $\frac{w}{w_n}$

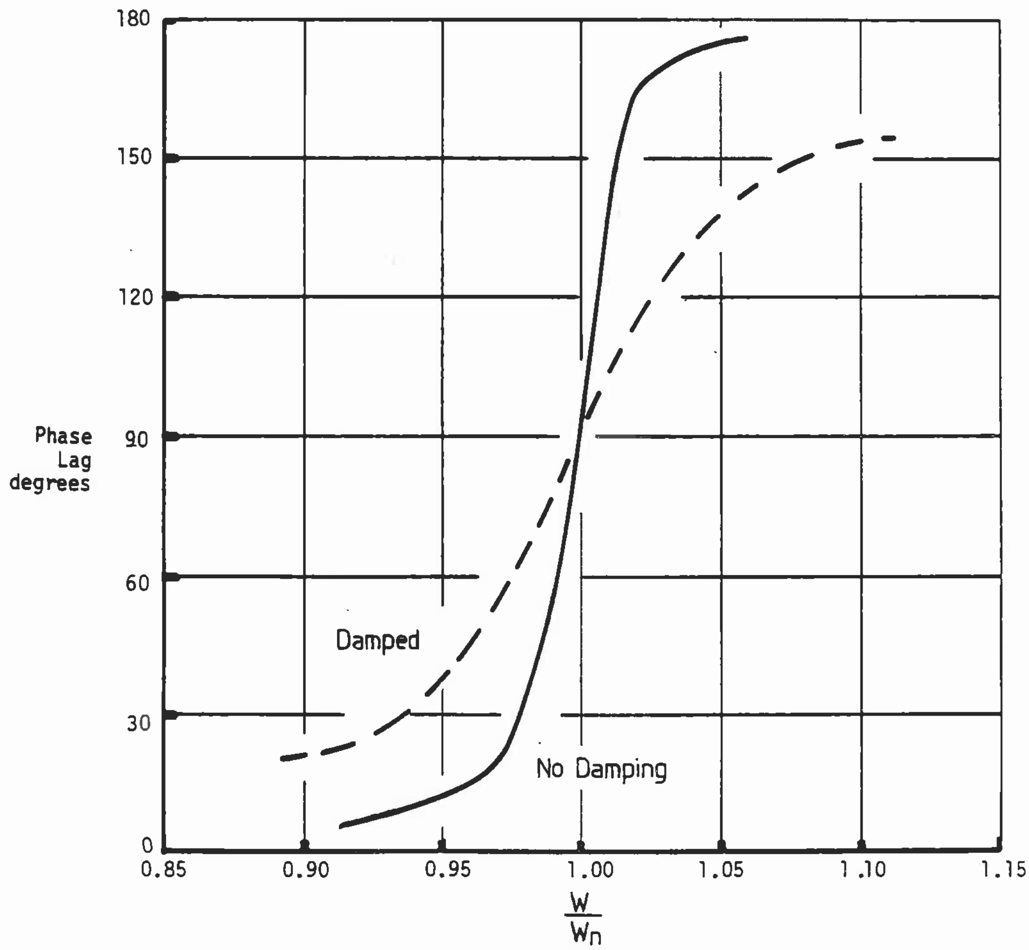


Fig. 14.4. Phase lag against ratio $\frac{w}{w_n}$

SECTION FOUR: REFERENCES

1. J HANNAH and R.C. STEPHENS: "Mechanics of Machines - Advanced Theory and Examples", Edward Arnold 1972 London

2. W. T. THOMPSON: "Theory of Vibration" Allen & Unwin 1981 London.

3. W. W. SETO "Mechanical Vibrations",Schaum Outline Series, McGraw-Hill, 1964 USA.

4. J. L. MERIAM: "Dynamics - 2nd Edition", John Wiley 1971 USA.

5. J. P. DEN HARTOG "Mechanical Vibrations", McGraw-Hill 1956 USA.

6. S. P. TIMOSHENKO: "Vibration Problems in Engineering", 4th Edition, Wiley 1974 W. Sussex, U.K.